# Kinds of Time

## 1. Aristotle's time

Aristotle says in the *Physics*: "Not only do we measure the movement by the time, but also the time by the movement, because they define each other. The time marks the movement, since it is its number, and the movement the time. We describe the time as much or little, measuring it by the movement, just as we know the number by what is numbered, e.g. the number of the horses by one horse as the unit. For we know how many horses there are by the use of the number; and again by using the one horse as unit we know the number of the horses itself. So it is with the time and the movement; for we measure the movement by the time and vice versa. It is natural that this should happen; for the movement goes with the distance and the time with the movement, because they are quanta and continuous and divisible. The movement has these attributes because the distance is of this nature, and the time has them because of the movement. And we measure both the distance by the movement and the movement by the distance; for we say that the road is long, if the [movement of the] journey is long, and that this [the movement of the journey] is long, if the road is long – the time [is long], too, if the movement [of the journey is long], and the movement [of the journey is long], if the time [is long]." (translated by R. P. Hardie and R. K. Gaye).

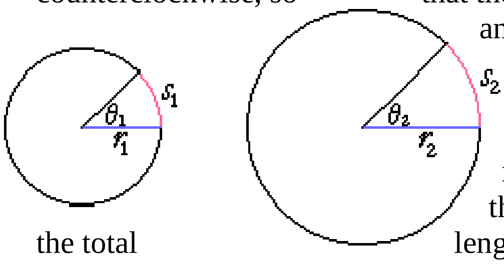
In talking about time measured (or "numbered") by motion, I will assume that changes of position of visible objects, i.e. distances between them, can be measured without paying any attention to questions about how long it may take to perform acts of measuring. I will take motion to be measured by changes of distances of a long hand of one common kind of clock. *Since motion is in this case measured without taking into account how much some sort of "time" elapses, motion is* measured *"atemporally".*


There are clocks which have circular faces near the circumference of which numerals from 1 to 12 are inscribed, and which have two hands attached to the center of the circular face which are rotated by some mechanism. The mechanisms are intended to move the hands with a regularity which approximates that of the sun's motion relative to other stars and to positions on the surface of the earth. Sundials may be taken to be ancestors of such clocks. [1]

*The motions of celestial objects with respect to terrestrial observers can also be* measured *"atemporally".* This is done by astronomers when they define right ascension and declination by setting an hour to be 15˚ of arc along certain circles on an imaginary celestial sphere. I will show that degrees of arc traversed along the circumference of a particular circle can be defined "atemporally".

A *sector* of a circular disk is the part of the disk bounded by two radii and an included arc, as shown in the figures below. The two endpoints of the included arc may be taken to mark positions of objects, whose distances apart measured along the included arc are different for different sized circles. For example, one of the disks may be taken to represent the face of a clock, and the other a disk determined by an observer at the center of the disk who measures the angle between two positions of a celestial object along an imaginary celestial sphere centered at the observer's position. This is a crude approximation to actual motion of, say, the sun, but it is consonant with the astronomical techniques known to Aristotle. For example, it doesn't take into account the elliptical motion of the earth's orbit (itself subject to perturbations), and it ignores the fact that the observer is not at the center of the earth but on the earth's rotating surface. However, my point is not to establish how time is measured nowadays, but to give an interpretation Aristotle's explanation of how "time" is the measure ("number") of motion, without considering the many mysteries which are said to be characteristic of "time". Aristotle did not have clocks of the sort illustrated above, but one may visualize instead a sundial, an ancestor of such clocks. A sundial may be thought of as a kind of clock without a mechanism to turn the hands, which instead makes use of shadows.

Given a circle with length of circumference C and length of any one of its radii r (same units of length as C), a radius may be rotated about the center of the circle, say counterclockwise, so that the new position and old position of the radius form an angle. The angle formed by two such positions of a radius will determine an arc of length s on the circumference. The angle is then said to be s/r radians in size, or more simply, since s/r is just a number without physical dimension, the measure of the angle is said to be s/r radians. The ratio C/r of the total length of the circumference C of a circle to the length of its radius r is $2\pi$. Thus the angle formed by one complete revolution of a radius is $2\pi$ radians = 360˚. Given two circles with radii $r_1$ and $r_2$ which when rotated cut off arcs $s_1$ and $s_2$ in such a way that $s_1/r_1 = s_2/r_2$, the angles formed in the two circles will be equal in size, i.e. in the above figure $\theta_1 = \theta_2$.




This if the two circular disks in the figure represent a clock and an astronomical observation of the sun's motion from the earth, one may suppose that the mechanism of the clock is made in such a way that the clock's hands trace out the same size angles as the sun's observed positions do. More simply, if the smaller circle represents a sundial, then the tips of the shadows cast by the gnomon will necessarily trace out the same size angles. (The *gnomon* is the upright part of the sundial; see the illustration of a sundial above.)

Consider such a circular clock with length of circumference C and length of radius r. Successively rotate the longer hand of this clock 60 times through an angle of $2\pi/60$ radians = 6˚. Let s denote the length of arc cut off on the circumference by a radius formed by the long hand during one of these rotations. Then for each such rotation there is an observable change of position of the hand, measured by the angle formed by the old and new positions of the hand. The size of this angle is s/r radians.

I will now introduce some terms which are commonly used to designate units of "time", but defined in terms of angular measure rather than in some other way. Set 1 H = $2\pi = 360^\circ$, so that the angle for one complete rotation measures 1 H. Also define a unit M by 1 M = H/60 = $2\pi/60 = 6^\circ$, so that 60 M = 1 H.

Suppose, for example, a vehicle travels 90 km, and during the trip the hand of a particular clock rotates through an angle measuring $3\pi$ radians = $540^\circ$. Set H = $2\pi$ radians = $360^\circ$, which measures the angle swept out during one revolution of a long hand or a gnomon's shadow on our clock. Then the average *rate* at which the vehicle traveled may be said to be 90 km/1.5H = 60 km/H = 60 km/$2\pi$ radians = 30 km/$\pi$ radians. This may be read as 30 km per each angular change of size $\pi$ radians = $180^\circ$, or if preferred, as 60 km per each complete rotation of the longer hand of the clock being used.

Suppose we call each revolution H, measured in radians, an *hour*, and each H/60 = M a *minute*. That is, suppose we call $2\pi$ radians or $360^\circ$ an *hour* and $2\pi/60$ radians or $6^\circ$ a *minute*, so that there are 60 minutes in an hour. In the example above, we can then *say* that the average rate of change, or *speed*, of the vehicle is 60 km/hour.

Suppose further that we understand these hours and minutes *only* as referring to angular measures as determined in the way described above, and *not* as referring to something called "time", or "passage of time", or "consciousness of time", etc/. Then we have defined rates of change of distances, or speeds, *not* as so many units of distance per units of some entity called "time" which flows or passes or which people pass in or experience, but as so many units of distance per revolutions of a line segment, such as the long hand of a kind of clock.

Suppose we go further and *define* "time" in this way. That is, suppose we say that what "time" *is*, is a measure of fractions of revolutions of the radii of circle. We then have, as Aristotle proposed, "time" as a *measure* or "number" of motion, where motion is itself measured with units of distance. Conversely, if we have "time" *defined* this way, and speeds defined by units of distance per units of this kind of "time" then we can calculate the corresponding changes in distance. For example, if we know that a vehicle travels at an average speed of 60 km/hour (i.e. 60 km per $2\pi$ radians) for 1.5 hours (i.e. for $3\pi$ radians), then it will traverse a distance of 90 km.

So "time," defined this way, is a number or measure of motion, and motion is a number or measure of "time," as Aristotle said.

## 2. Newton's time.

Isaac Newton writes in the Scholium to the Definitions in his *Philosophiae Naturalis Principia Mathematica* (translated by Andrew Motte, revised by Florian Cajori.):

“V. Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time. For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time; astronomers correct this inequality that they may measure the celestial motions by a more accurate time.”

Comment 1. From Wikipedia at http://en.wikipedia.org/wiki/Equation_of_time:

“The **equation of time** is the difference, over the course of a year, between time as read from a sundial and a clock. The sundial can be ahead (fast) by as much as 16 min 33 s (around November 3) or fall behind by as much as 14 min 6 s (around February 12). It is caused by irregularity in the path of the Sun across the sky, due to a combination of the obliquity of the Earth's rotation axis and the eccentricity of its orbit.”

“It may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change.”

Comment 2. I take it that Newton means here that there might be no two (or more) *observable* motions which can be compared without some discrepancy arising. I take it also that to speak of absolute time as *flowing* amounts to saying that such time moves, i.e. absolute time is a kind of motion, though not a change of position in space. This motion, he prescribes, is to be taken as not itself moving (see Comment 4 below). The terms “acceleration” and “retardation” are customarily defined as change in velocity, where velocity is change of distance per unit of some kind of time – *but not Newton’s absolute time*. Here, however, “acceleration” or “retardation” of absolute time may be taken to refer to some kind of change in the way the universe is changing, which Newton assumes does not happen.

“The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation. The necessity of this equation, for determining the times of a phenomenon, is evinced as well from the experiments of the pendulum clock, as by eclipses of the satellites of Jupiter.”

Comment 3. Absolute time is then “duration or perseverance of the existence of things”, a kind of motion peculiar to existence or being. This kind of motion, says Newton, can be measured by comparing certain kinds of observable *relative* motions, observations of observable things in space, such as movements of shadows cast by (apparent) movements of our Sun with respect to some position on our Earth, or movements of hands of a pendulum clock compared with observable astronomical movements.

"VI. As the order of the parts of time is immutable, so also is the order of the parts of space. Suppose those parts to be moved out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the places as well of themselves as of all other things. All things are placed in time as to order of succession; and in space as to order of situation. It is from their essence or nature that they are places; and that the primary places of things should be movable, is absurd. These are therefore the absolute places; and translations out of those places, are the only absolute motions."

> Comment 4. Thus absolute time is a kind of motion which occurs in a kind of *place*, analogous to a place in space. The nature of this absolute motion is such that when it is measured by using relative motions, one state of absolute time follows another so as to form a succession, a kind of linear ordering, as evidenced by swings of a pendulum, or hands of a mechanical clock, or movements of shadows cast by the gnomon of a sundial. This succession is a kind of *translation*, which may be represented mathematically by visualizing a straight line in the manner of Euclid, in which points are ordered and labeled, and placed so that the distance between two successive integers is always the same, and are interpreted as units of time, e.g. seconds, days, years. [2]

So much for Newton's two kinds of time, absolute and relative times. Relative times are measures of absolute time. In terms of subjectivity and objectivity, Newton's absolute time is an *objective* time. No matter by whom or how it is measured, the measurement leads to a mathematical representation of the sort just described. Absolute time is invariant under change of an observer who measures. However, equations of time and choices of position must be used to transform one observer's measurements into another observer's measurements. Newton's relative times are *subjective*, since they are made and interpreted by different observers using different methods and different kinds of equipment, and they are subject to certain corrections and correlations when they are used to measure the flow of absolute time in its place. Newton's absolute time and elative times since he postulates that any suitable measurements of certain *periodic* relative motions (i.e., relative times), such as for example swings of a pendulum compared with apparent movements of the sun, may be used, when suitably transformed, to give a measure (at least to a useful approximation) of absolute time. Since absolute time flows, it is a kind of motion, which I will call *absolute motion*,

Thus it appears that there is a resemblance between Aristotle's proposal that time is the number (measure) of motion, and Newton's proposal that relative motions are measures of absolute motion. However, Newton's "time" defined as a motion is not described by him as a measure of relative times.

### 3. Einstein's argument.

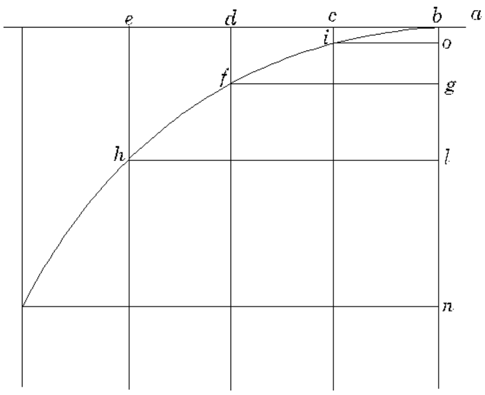
[1] In her book *What, Then, Is Time?* (1999) Eva Brann writes: "Again, when it [time] is spoken of in anthropology, for example, when the Mayan "concept of time" is presented, the object of study is an intricate and significance-bearing calendrical system, and I have not come across the claim that the Mayans even had a separate word for the notion of time . . . they seem to have used the word for "sun"."

[2] Michael Fowler, a physicist, gives the following at
http://galileoandeinstein.physics.virginia.edu/tns244.htm

"*Beginning on page 244 of Two New Sciences, Galileo gives his classic analysis of the motion of a projectile as a compound motion, made up of a horizontal motion which has steady speed in a fixed direction, and a vertical motion which is his "naturally accelerated motion" picking up velocity in the downward direction at a steady rate.*

[Fowler quotes from Galileo's *Dialogue Concerning Two Sciences* as translated by Henry Crew and Alfonso de Salvio, Macmillan, 1914]:

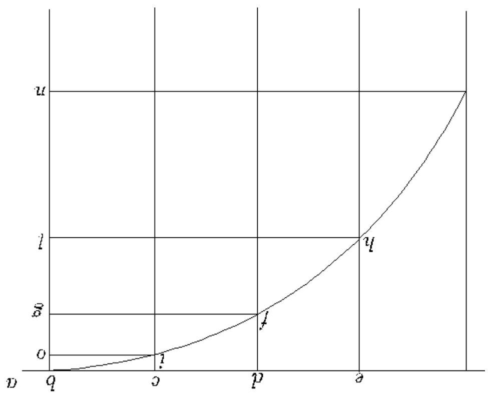
Let us imagine an elevated horizontal line or plane *ab* along which a body moves with uniform speed from *a* to *b.* Suppose this plane to end abruptly at *b*; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular *bn*.




Draw the line *be* along the plane *ba* to represent the flow, or measure, of time; divide this line into a number of segments, *bc, cd, de,* representing equal intervals of time; from the points *b, c, d, e,* let fall lines which are parallel to the perpendicular *bn*. On the first of these lay off any distance *ci*, on the second a distance four times as long, *df;* on the third, one nine times as long, *eh;* and so on, in proportion to the squares of *cb, db, eb,* or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from *b* to *c* with uniform speed, it also falls perpendicularly through the distance *ci,* and at the end of the time-interval *bc* finds itself at the point *i*. In like manner at the end of the time-interval *bd,* which is the double of *bc,* the vertical fall will be four times the first distance *ci*; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space *eh* traversed during the time *be* will be nine times *ci*; thus it is evident that the distances *eh, df, cl* will be to one another as the squares of the lines *be, bd, bc.* Now from the points *i, f, h* draw the straight lines *io, fg, hl* parallel to *be;* these lines *hl, fg, io* are equal to *eb, db* and *cb*, respectively; so also are the lines *bo*, *bg*, *bl* respectively equal to *ci*, *df*, and *eh.* The square of *hl* is to that of *fg* as the line *lb* is to *bg;* and the square of *fg* is to that of *io* as *gb* is to *bo*; therefore the points *i, f, h,* lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola. Q. E. D.

[Fowler adds:] *(Note that in the above, Galileo is using the horizontal intervals bc, cd, etc., to denote time as well as distance. This is ok, since the horizontal motion is at steady speed, but is rather confusing!)*"

If we take Galileo's picture illustrating his law of falling bodies, and rotate it to the left 180˚, we get:



If we remove the upside-down letters, label the left vertical line with an "s" (or an "x") and the bottom horizontal line with a "t", and insert some numbers appropriately where the letters were, we will get a diagram of the sort we customarily see nowadays.

In connection with timing, I note that Galileo used his pulse as a kind of timing device, rather than a clock of a kind we now have available.  Concerning how bodies accelerate when they fall toward the earth, one of the participants in his *Dialogue Concerning Two New Sciences* says:

"When I think of a heavy body falling from rest, that is, starting with zero speed and gaining speed in proportion to the time from the beginning of the motion; such a motion as would, for instance, in eight beats of the pulse acquire eight degrees of speed; having at the end of the fourth beat acquired four degrees; at the end of the second, two; at the end of the first, one: and since time is divisible without limit, it follows from all these considerations that if the earlier speed of a body is less than its present speed in a constant ratio, then there is no degree of speed however small (or, one may say, no degree of slowness however great) with which we may not find this body travelling after starting from infinite slowness, i. e., from rest. So that if that speed which it had at the end of the fourth beat was such that, if kept uniform, the body would traverse two miles in an hour, and if keeping the speed which it had at the end of the second beat, it would traverse one mile an hour, we must infer that, as the instant of starting is more and more nearly approached, the body moves so slowly that, if it kept on moving at this rate, it would not traverse a mile in an hour, or in a day, or in a year or in a thousand years; indeed, it would not traverse a span in an even greater time; a phenomenon which baffles the imagination, while our senses show us that a heavy falling body suddenly acquires great speed."  (translated by Crew and de Salvio)